Wealth and the Principal-Agent Matching

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Motivation

- In the classical moral hazard literature, cash constraints for the agent might have a significant role on written contracts and efficiency.
- Literature about matching between principals and agents has focused on supermodularity assumptions, and on talent, but mainly ignoring moral hazard, and, if they do account for it, they neglect the agent's limited liability.
- What role does wealth play in the matching if the moral hazard problem is taken into account?

Literature Review

1. Wealth on moral hazard:
    1. Thiele and Wambach (JET 1999), Chade and de Serio (G&EB 2014).
    2. Dam and Pérez-Castrillo (FTE 2006).
2. Empirical papers:
    1. Ackerberg and Botticini (JPE 2002).
    2. Baker and Hall (JLE 2004).
    3. Gabaix and Landier (QJE 2008), Tervio (AER 2008).
    4. Edmans, Gabaix, and Landier (RFS 2009).
    5. Bandiera, Guiso, Prat and Sadun (JLE 2008).

Steps

1. Describe the isolated principal-agent model, and the role of limited liability.
2. Use non transferable utility to study the matching between principals and agents.
3. Study the interaction between wealth and talent in the matching with principals.

Limited Liability

Why is wealth relevant? It's not hard either to come up with a few examples in which wealth is relevant:

- Franchising model.
- Entrepreneurship.
- Partnerships.

Principal-Agent Model

Standard moral hazard model:

1. A risk-neutral principal owns a firm of size $\xi \in (0, 1]$.
2. She hires a risk neutral agent to run it. He can exert effort $e \in [0, 1]$, which cannot be observed by the principal.
3. Agent succeeds ($x = \xi$) with probability e, or fails ($x = 0$) with probability $1 - e$.
4. Effort is costly for the agent, $c(e) = \frac{e^2}{2\tau}$. $\tau \in (0, 1]$ represents the agent's talent.
5. The agent has personal wealth ω.

Principal-Agent Model

The principal proposes a take-it-or-leave it contract:

1. Fixed wage a.
2. Bonus in case of success b.

that solves the following problem:

$$\max_{e,a,b} \quad -a + e(\xi - b) \tag{1}$$

$$s.t. \quad e \in \arg\max_{\hat{e}} \quad \left\{ a + \hat{e}b - \frac{\hat{e}^2}{2\tau} \right\} \tag{IC}$$

$$a + eb - \frac{e^2}{2\tau} \geq \overline{u} \tag{PC}$$

$$a \geq -\omega \tag{CC}$$

Principal-Agent Model

The solution to this problem depends on the value of ω and ξ:

	Binding Constraint		
Variable	(CC) $\omega + \overline{u} < \frac{\xi^2\tau}{8}$	(CC) and (PC) $\frac{\xi^2\tau}{8} \leq \omega + \overline{u} \leq \frac{\xi^2\tau}{2}$	(PC) $\frac{\xi^2\tau}{2} < \omega + \overline{u}$
e	$\frac{\xi\tau}{2}$	$\sqrt{(\overline{u}+\omega)2\tau}$	$\xi\tau$
a	$-\omega$	$-\omega$	$\overline{u} - \frac{\xi^2\tau}{2}$
b	$\frac{\xi}{2}$	$\frac{\sqrt{(\overline{u}+\omega)2\tau}}{\tau}$	ξ
$E[u]$	$\frac{\xi^2\tau}{8}$	$\overline{u}+\omega$	$\overline{u}+\omega$
$E[\Delta u]$	$-\omega + \frac{\xi^2\tau}{8}$	$\overline{u}$	$\overline{u}$
$E[v]$	$\omega + \frac{\xi^2\tau}{4}$	$\xi\sqrt{(\overline{u}+\omega)2\tau} - 2\overline{u} - \omega$	$\frac{\xi^2\tau}{2} - \overline{u}$
$E[\Delta Surplus]$	$\frac{3\xi^2\tau}{8}$	$\xi\sqrt{(\overline{u}+\omega)2\tau} - \overline{u} - \omega$	$\frac{\xi^2\tau}{2}$

Table: Solution to the maximization problem of the agent.

Principal-Agent Model

Summarizing, in the one-to-one model:

1. $\uparrow \omega$, agent is cash constrained only for bigger firms.
2. $\uparrow \xi$, richer agents will become cash constrained for the firm.
3. If the agent is cash constrained, his information rents decrease with ω.
4. Firm profits increase with ω for $\omega < \frac{\xi^2 \tau}{2}$ and are constant for values equal or above.

Non perfectly transferable utility

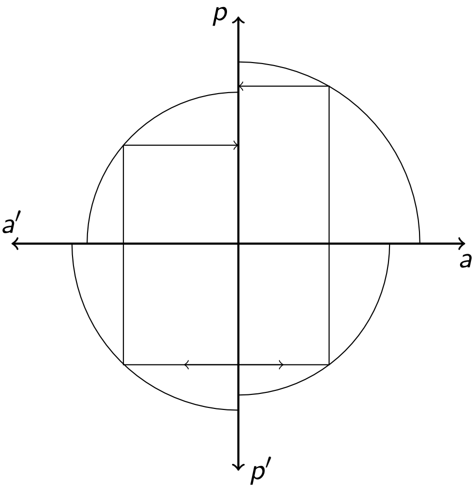
I assume that the number of agents is at least as high as the number of principals, and therefore the **bargaining power remains with the principal**.

- Classical matching theory mainly uses two extreme cases:
  - Economic agents can transfer utility freely among them, or,
  - Any transfer of utility among them is forbidden.
- The principal can transfer utility to the agent by setting a higher fixed wage (a), however, doing so by increasing the bonus (b) is more efficient, because of the change on incentives. The transference is not 1-to-1.

Legros and Newman [LN, Econometrica 2007] define generalized increasing differences (GID) to deal with the whole spectrum of utility transfers.

Non perfectly transferable utility

Graphical representation of GID, and the importance of the utility possibility frontier (UPF).



Utility Possibility Frontier

Let $\theta = (\omega, \tau)$ be the type of the agent.

Definition 1

The UPF is described by the following functions:

- Let $\phi(\xi, \theta, u)$ be the maximum utility that a principal type ξ can obtain by matching with an agent type θ, and this agent gets utility u.
- Let $\psi(\theta, \xi, v)$ be the maximum utility that an agent type θ can obtain by matching with a principal type ξ, and this principal gets utility v.

Utility Possibility Frontier

Definition 2

Given the UPF, define:

- $\underline{u}_\theta(\xi) := \max\{0, \frac{\xi^2\tau}{8} - \omega\}$. This is the minimum level of utility an agent type θ can get from a matching with a principal type ξ.
- $V_\xi(\theta) := [0, \phi(\xi, \theta, \underline{u}_\theta(\xi))]$ feasible utility levels that an optimal contract can give to a principal whose firm has size ξ contracting with an agent type θ. By definition, $V_\xi(\theta)$ is also the domain of $\psi(\theta, \xi, \cdot)$.
- $U_\xi(\theta) := [\underline{u}_\theta(\xi), \frac{\xi^2\tau}{2}]$ set of feasible utility levels that an optimal contract can give to an agent type θ working for a firm of size ξ. By definition, $U_\xi(\theta)$ is also the domain of $\phi(\xi, \theta, \cdot)$.

Utility Possibility Frontier

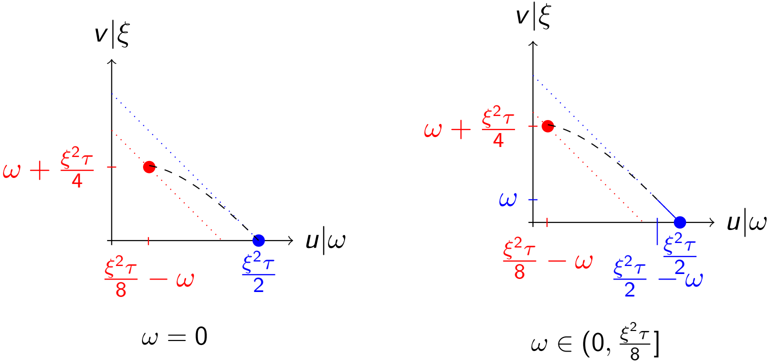
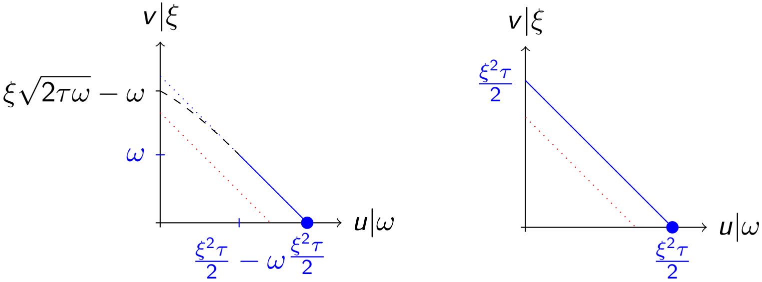


Figure: UPF for different levels of ω.

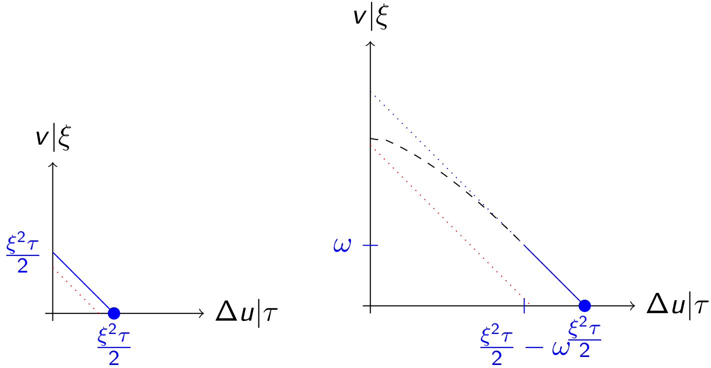
Utility Possibility Frontier



$\omega \in (\frac{\xi^2\tau}{8}, \frac{\xi^2\tau}{2}]$ $\omega \in (\frac{\xi^2\tau}{2}, \infty)$

Figure: UPF for different levels of ω.

Utility Possibility Frontier



(a) $\tau \in \left[0, \frac{2\omega}{\xi^2}\right]$ (b) $\tau \in \left[\frac{2\omega}{\xi^2}, \frac{8\omega}{\xi^2}\right]$

Figure: UPF for different levels of τ.

Utility Possibility Frontier

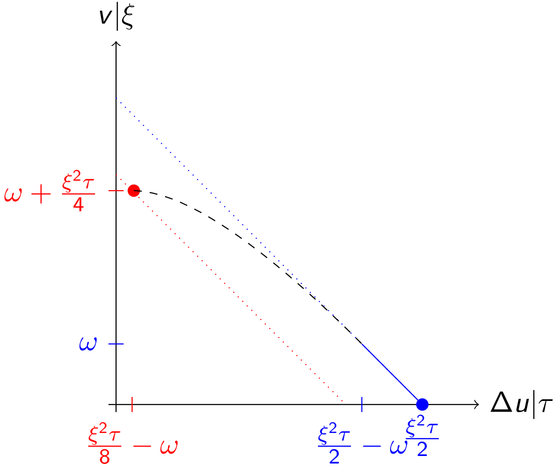


Figure: $\tau \in \left[\frac{8\omega}{\xi^2}, \infty\right]$

Matching

θ and ξ

Fact 3

If talent is homogeneous among the agents, and they are wealthy enough such that they are not cash constrained for any firm, then no type of matching can be ruled out.

Matching

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If talent is homogeneous among the agents, and they are wealthy enough such that they are not cash constrained for any firm, then no type of matching can be ruled out.

In this case, the principal can always "sell the firm" to the agent, achieving first best.

Matching

θ and ξ

Lemma 4

The UPF described by $\phi(\xi, \theta, u)$ is:

- *Continuous and strictly decreasing in u for $u \in U_\xi(\theta)$.*
- *Differentiable in u for $u \in int(U_\xi(\theta))$.*
- *Differentiable in ξ.*

Matching

θ and ξ

Lemma 5

$\partial\phi/\partial\xi$ is continuous and increasing in τ, ω, and u. It is also piece-wise differentiable in each of these variables.

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Lemma 6

$\phi(\xi, \theta, u)$ and $\psi(\theta, \xi, v)$ are type increasing, that is:

- *ϕ is non decreasing in ω and τ.*
- *ψ is non decreasing in ξ.*

Matching

θ and ξ

Proposition 1

The economy with moral hazard satisfies generalized increasing differences in (ξ, ω) and (ξ, τ), which implies that:

- *For equally talented agents, larger firms will match with wealthier agents.*
- *For equally wealthy agents, larger firms will match with more talented agents.*

Matching

θ and ξ

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This means that positive assortative matching (PAM) will appear in this economy. This result holds for a continuous (or discrete) set of principals and agents. For simplicity, I will now focus in the case in which types are discrete.

Matching

θ and ξ

Proposition 2

Let $\theta = (\omega, \tau)$ and $\theta' = (\omega', \tau')$ be the types of two consecutive agents such that $\omega' \leq \omega$ and $\tau' \leq \tau$. One coordinate (τ or ω) is equal among all the agents, while the other (ω or τ) is strictly larger. The equilibrium outcome must satisfy:

- *If (θ', ξ') represents the match with the lowest matched types in the type set, agent and principal will obtain:*

$$u^* = \underline{u}_{\theta'}(\xi') \qquad v^* = \phi(\xi', \theta', u^*)$$

- *Otherwise, let $\tilde{v}$ be the utility of the low type principal when preventing being outbid for her matched agent. The high type match will obtain:*

$$u^* = max\{\psi(\theta, \xi', \tilde{v}), \underline{u}_{\theta}(\xi)\} \qquad v^* = \phi(\xi, \theta, u^*)$$

Matching

θ and ξ

Proposition 3

Consider consecutive matches of firms of size $\xi' < \xi$, and agents with types $\theta' < \theta$, such that only ω or τ is equal between them, and for every match, the agents are cash constrained for both firms.
The high type match will write a contract with stronger incentives than the contract they would write out of the market, and therefore closer to the first best output if:

- $\omega - \omega' > 2\xi'^4\tau + \frac{(\xi^2-\xi'^2)\tau}{4} - \xi'^2\tau\sqrt{\xi^2 - \xi'^2 + 4\xi'^4}$, *when the agents' type is wealth.*
- $\frac{\tau'}{\tau} < 2 - \frac{\xi^2}{2\xi'^2}$, *when the agents' type is talent.*

Matching

The previous proposition accounts for the competition pressure. The closer the types are, the more pressure will the high type principals have to face to provide more incentives to their agents.

This would explain the empirical result (Tervio 2008) in which a concentrated distribution of talent exhibits a disperse distribution for compensation. Even further, the model provides the tools to disentangle and measure how much of compensation comes from the competition pressure.

Bidimensional Matching

The theory of multidimensional matching is being developed in this very moment, e.g. Chiappori (2016).
Nevertheless, it is still possible to study, by looking for GID or DID, the effects of wealth and talent in the matching. I provide an example.

Bidimensional Matching

A simple extreme case:

- Let there be 2 agents and 2 principals.
- Let (τ', ω) and $(\tau, 0)$ be the endowments of each agent. With $\tau' < \tau$ and $0 < \omega$.
- Let $\xi' < \xi$ be the two types of principals.
- Let ξ and ω be such that the agent who has wealth ω is not cash constrained for either firm.

Bidimensional Matching

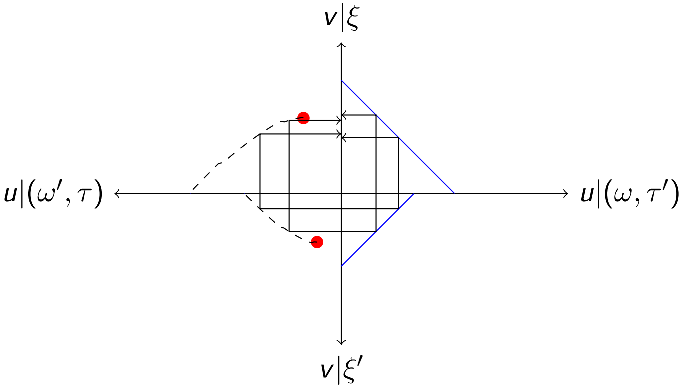

Figure: $\tau - \tau' < \underline{\tau}$.

Bidimensional Matching

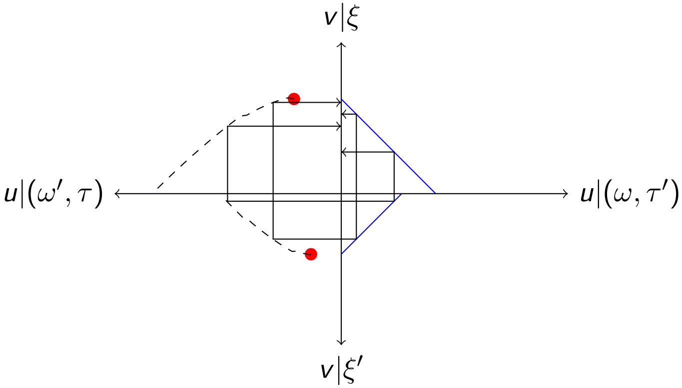

Figure: $\tau - \tau' \geq \underline{\tau}$.

Bidimensional Matching

From what can be observed:

- If talent is relatively similar among agents ($\tau - \tau' \leq \overline{\tau}$), then the effect of dissimilar wealth can actually dominate and break the conditions to have PAM in talent.
- However, if talent is very different among agents ($\tau - \tau' \geq \overline{\tau}$), then this effect dominates and PAM with respect to talent will arise.

Wealth can play a significant role when talent is not very dispersed. Considering that the correlation between wealth and talent may not be strong enough when talent is relatively similar, empirical studies should put a lot of attention on ω before neglecting it, or using it as a proxy for risk aversion.

Conclusions

1. The model points to endogenous PAM between principal and agents, when the types are firm size, and talent or wealth respectively.
2. Competition can make higher types matches to establish compensation for agents that go beyond what would happen in absence of the market.
3. More homogeneous environments will create a higher competition pressure.
4. If talent is not very dispersed among agents, then wealth can break the traditional PAM between firm size and agent's talent.